1 Introduction

This paper investigates and compares Aristotle and Boethius on syllogisms. First, we will recall Aristotle’s ideas on categorical syllogisms followed by his ideas on hypothetical syllogism (HS, afterwards). I will, thereafter, follow the same organizational path for Boethius. In order to keep focused, modal syllogisms will be excluded from our investigation.

Let me first remind you what syllogisms and hypothetical syllogisms are. A categorical syllogism is a deduction consisting of two premises and one conclusion. One of the most common examples is given as follows:

All men are mortal.
All mortals are alive.
All men are alive.

Aristotle’s own definition of syllogism from Prior Analytics is as follows:

[A syllogism is] a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last phrase that they produce the consequence, and by this, that no further term is required from without to make the consequence necessary. ([1], Book I, 24a, 58)

It has been noted by Rusinoff in [11] that this definition of Aristotle does not distinguish syllogisms from other forms of inference. Yet, Aristotle in Prior Analytics ([1], Book I, 25b, 27) distinguished syllogisms from demonstration. Aristotle wrote,

Sylogism should be discussed before demonstration, because syllogism is the more general: the demonstration is a sort of syllogism, but not every syllogism is a demonstration.
In other words, as Corcoran put it for Aristotle, “a demonstration is an extended argumentation that begins with premises known to be truths and involves a chain of reasoning showing by deductively evident steps that its conclusion is consequence of its premises” [3]. In other words, “a demonstration reduces a problem to be solved to problems already solved” [3].

On the other hand, hypothetical syllogisms are those where at least one premise is a hypothetical sentence. Following this definition, we can come up with an example. For instance, the following are both hypothetical syllogisms.

\[
\begin{align*}
\text{If } A \text{ is, } B \text{ is} \\
\text{If } B \text{ is, } C \text{ is} \\
\text{If } A \text{ is, } C \text{ is} \\
\text{But } A \text{ is} \\
\text{Therefore, } B \text{ is}
\end{align*}
\]

It can be remarked that, in the first syllogism, both premises are hypothetical sentences, whereas in the latter example, only the major premise is a hypothetical sentence.

As we will present in the next section, Aristotle gave the classification of syllogisms with respect to their forms. Kneale and Kneale in their Development of Logic claimed that this is due to Aristotle's interest in demonstrative sciences ([6], p. 67). That is, according to Kneale and Kneale, Aristotle basically followed the same methodology in the course of syllogisms by giving classifications, as he did, for instance, in biology and taxonomy. But, in any case, Aristotle's theory of syllogisms can be named as the "history's first serious attempt at a comprehensive theory of inference" ([12], p. 64).

Boethius, on the other hand, considered himself having the mission of translating and commenting on Aristotle's work, and tried to make Aristotle's works more explicit and clearer by his commentaries. For this purpose, Boethius also made use of the expositions of Peripatetic School on syllogisms. But, due to the difference in the languages (he translated from Greek to Latin), some obstacles occurred which we will explain in Section 3.

Therefore, in this paper, we will discuss the syllogisms from both fronts: Aristotle and Boethius. The topic is deep, so we will mainly focus on the differences with respect to categorical and hypothetical syllogisms in Aristotle and Boethius.

2 Aristotle on Syllogisms

2.1 Categorical Syllogisms in Aristotle

As an “achieved and completed body of doctrine”\(^2\), syllogisms first appeared in Aristotle's Prior Analytics. Aristotle considered the categorical statements, which consist of a quantifier, a subject term, a copula and a predicate term in which

\(^1\)Corcoran also mentions that Tarski implies that “formal proof in the modern sense results from a refinement and ‘formalisation' of traditional Aristotelian demonstration” [3]

\(^2\)Quoting Kant from [4], p. 180.
Can Başkent

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a copula connects the (quantified) subject to the predicate, and may be in the form of “was” or “is”.

As Corcoran noted, on the other hand, “categorical syllogism is the restricted system he created to illustrate” the deduction, and thus “Aristotle’s general theory of deduction must be distinguished” from it [3].

Aristotle classified categorical statements into four groups: universal affirmative (A), particular affirmative (I), universal negative (E), particular negative (O). Those four classes can be denoted as follows:

(A) A belongs to all B. (AaB)
(I) A belongs to some B. (AiB)
(E) A does not belong to any B. (AeB)
(O) A does not belong to some B. (AoB)

As we pointed out already a syllogism is a deduction consisting of two premises and one conclusion, all of which are categorical statements. So, there are three terms in a syllogism: major (meizon akron), minor (elatton akron) and the middle term (meson). Major and minor terms establish the predicate and subject of the conclusion whereas the middle term joins them.

These three terms might be combined to form the three schemata. If A is the major, B is the middle, and C is the minor term, the three schemata are given in the Table 2.1. The four categorical sentences (a, e, i, o), then, can be placed into the three given figures in the Table 2.1 to get the 14 valid moods that Aristotle ended up. The aforementioned 14 valid moods are given in the following tables Table 2, Table 3, and Table 4. Note that, all these tables are driven from [1].

Table 1: The Three Schemata

<table>
<thead>
<tr>
<th>I.</th>
<th>II.</th>
<th>III.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - B</td>
<td>B - A</td>
<td>A - B</td>
</tr>
<tr>
<td>B - C</td>
<td>B - C</td>
<td>C - B</td>
</tr>
<tr>
<td>A - C</td>
<td>A - C</td>
<td>A - C</td>
</tr>
</tbody>
</table>

However, a fourth schemata can also be added (see Table 5). The fourth figure was not explicitly given by Aristotle, but only pointed out by him according to Dumitriu ([4], p. 198). Dumitriu also noted that the fourth figure was developed by Aristotle’s students who formed Peripatetic School and directed Lyceum after him (see Table 6). Kneale and Kneale agreed with Dumitriu. They further indicated that ([6], p. 100) it was first Theophrastus who “added five moods which will be later form the fourth figure”. Dumitriu also added Eudemus’s

3Although the abbreviations (a, e, i, o) were introduced in medieval era after Boethius, we adopt them for notational convenience.
Table 2: The First Figure

<table>
<thead>
<tr>
<th>Barbara</th>
<th>Celarent</th>
<th>Darii</th>
<th>Ferio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AaB$</td>
<td>$AeB$</td>
<td>$AaB$</td>
<td>$AeB$</td>
</tr>
<tr>
<td>$BaC$</td>
<td>$BaC$</td>
<td>$BiC$</td>
<td>$BiC$</td>
</tr>
<tr>
<td>$AaC$</td>
<td>$AeC$</td>
<td>$AiC$</td>
<td>$AoC$</td>
</tr>
</tbody>
</table>

Table 3: The Second Figure

<table>
<thead>
<tr>
<th>Camestres</th>
<th>Cesare</th>
<th>Festino</th>
<th>Baroco</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BaA$</td>
<td>$BeA$</td>
<td>$BeA$</td>
<td>$BaA$</td>
</tr>
<tr>
<td>$BeC$</td>
<td>$BaC$</td>
<td>$BiC$</td>
<td>$BoC$</td>
</tr>
<tr>
<td>$AeC$</td>
<td>$AeC$</td>
<td>$AoC$</td>
<td>$AoC$</td>
</tr>
</tbody>
</table>

Table 4: The Third Figure

<table>
<thead>
<tr>
<th>Darapti</th>
<th>Felapton</th>
<th>Disamis</th>
<th>Datisi</th>
<th>Bocardo</th>
<th>Ferison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AaB$</td>
<td>$AeB$</td>
<td>$AiB$</td>
<td>$AaB$</td>
<td>$AoB$</td>
<td>$AeB$</td>
</tr>
<tr>
<td>$CaB$</td>
<td>$CaB$</td>
<td>$CaB$</td>
<td>$CiB$</td>
<td>$CaB$</td>
<td>$CiB$</td>
</tr>
<tr>
<td>$AiC$</td>
<td>$AoC$</td>
<td>$AiC$</td>
<td>$AiC$</td>
<td>$AoC$</td>
<td>$AoC$</td>
</tr>
</tbody>
</table>
name besides Theophrastus ([4], p. 209) as a developer of the fourth figure. But, Kneale and Kneale also remarked that some Aristotle commentators (such as Alexander of Aphrodisias) thought that Theophrastus only made clear what Aristotle wrote in Prior Analytics ([6], p. 100). However, we think that who came up with the additional five moods to form the fourth figure first is not so clear considering that none of the works of Theophrastus survived. Therefore, we refrain ourselves from making a sharp comment on the origin of the fourth figure. 4

<table>
<thead>
<tr>
<th>Table 5: The Fourth Schemata</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI.</td>
</tr>
<tr>
<td>$B \cdot A$</td>
</tr>
<tr>
<td>$C \cdot B$</td>
</tr>
<tr>
<td>$A \cdot C$</td>
</tr>
</tbody>
</table>

As given in the tables, we have ended up with 19 valid moods. However, we can add two subalternate moods (Barbari and Celaront) to the first figure, two subalternate moods (Camestrop and Cesaro) to the second figure, and one subalternate mood (Camenop) to the fourth figure. We recall that, two particular statements (i) and (o) are said to be subaltern to universal statements (a) and (e) respectively. But, note that this terminology is not due to Aristotle himself ([6], p. 56).

Aristotle, among the three figures he gave, took the first one (Figure 2) and kept it as the set of axioms, and then proved the remaining three figures by reducing them to the first figure. Because, for him, nothings needs to be added to make the first figure evident and, he called it complete. Aristotle introduced three conversion rules in Prior Analytics ([11], Book I, 50b, 5 - 51b, 2) to be used in these reduction proofs. By modern notation, we can denote these reduction rules as follows.

- $AaB \Rightarrow BiA$
- $AiB \equiv BiA$

---

4There have been many arguments why fourth figure was excluded by Aristotle himself. A detailed discussion can be found in [10].
• $AeB \equiv BeA$.

What do they mean then? The first rule says, from a universal affirmative, a particular affirmative follows with the subject and the predicate swapped. The second and the third rules say, in the case of the particular affirmatives and the universal negatives, the subject and the predicate can be interchanged without any possible change in the meaning.

### 2.2 Hypothetical Syllogisms in Aristotle

Dumitriu remarked in ([4], pp. 182-3) that, Aristotle only pointed out HS in the *Organon*. Let us follow from *Prior Analytics* [1]:

It is possible for the premises of the syllogism to be true or to be false, or to be the one true, the other false. The conclusion is either true or false necessarily. From true premises it is not possible to draw a false conclusion, but a true conclusion may be drawn from false premises, true however only in respect to the fact, not to the reason.

(Book II, 53b, 4)

(...)

If it is necessary that $B$ should be when $A$ is, it is necessary that $A$ should not be when $B$ is not.

(Book II, 53b, 12)

(...)

(...) [It is impossible that $B$ should necessarily be great since $A$ is white and that $B$ should necessarily be great since $A$ is not white. For whenever since this, $A$, is white it is necessary that that, $B$, should be great, and since $B$ is great that $C$ should not be white, then it is necessary if is white that $C$ should not be white.

(Book II, 57b, 1)

These statement are equivalent to the following statements according to Dumitriu ([4], pp.182-3):

1. From true premises, one cannot draw a false conclusion, but from false premises one can draw a true conclusion.

2. If when $A$ is, $B$ must be, then when $B$ is not, necessarily $A$ cannot be.

3. If from $A$ follows necessarily $B$, and from $B$ follows non-$C$, then necessarily from $A$ follows non-$C$. 

6
The first argument basically depends on the truth table of implication. When both premises are true, a false conclusion cannot follow. However, a true conclusion follows from the false assumptions. The second argument gives the contrapositive of an implication, that is $A \rightarrow B$ is equivalent to $\neg B \rightarrow \neg A$. This fact also is based on the truth table of the implication. The third rule manifests the transitivity of hypothetical statements, i.e. $A \rightarrow B$ and $B \rightarrow C$ imply $A \rightarrow C$. This is a simple theorem of propositional calculus and can be obtained by an application of its basic rules.

However, these vague clues do not suffice to claim that Aristotle himself studied the theory of HS. Kneale and Kneale pointed out that “Aristotle did not recognize the conditional form of statement and argument based on it as an object of logical inquiry” ([6], p.98). Hypothetical syllogisms were first studied extensively by the Peripatetic School, and for Dumitriu, it is the most important contribution of Peripatetics ([4], p. 210).

Theophrastus, Aristotle’s successor as the director of Lyceum, divided them into two categories. The first group consisted of the HSs with two hypothetical statements for the major and the middle term, whereas the second group consisted of the HSs with one hypothetical statement either for the major or the middle term.

1. Shows the conditions under which something is or not. For example,

   If $A$ is, $B$ is
   If $B$ is, $C$ is
   If $A$ is, $C$ is

2. Expresses whether something does or does not exist. For example,

   If $A$ is, $B$ is
   But $A$ is
   Therefore, $B$ is

Thus, the basic resource for Boethius on the HS was not Aristotle himself, but the Peripatetic School.

3 Boethius on Syllogisms

Anicius Manlius Severinus Boethius is best known for his commentaries on Aristotle’s works. He devoted most of his time and intellectual energy to his big project of translation from the original Greek to Latin and composing philosophical commentaries on these translations. He translated all Aristotle’s work on logic, although as Marenbon stated ([8], p. 165), the translation of “Posterior Analytics did not survive into the middle ages”.

His interest in logic was not restricted only to translating Aristotle’s work and commenting on them. He also wrote a commentary on Porphyry’s Isagoge,
which was, according to Marenbon, the standard introduction to logic in Neo-platonic schools ([8], p. 165). More interestingly, he wrote a series of logical textbooks among which only five survived.

### 3.1 Categorical Syllogisms in Boethius

One of the first differences that Boethius had on his commentaries on Aristotle’s works is his construction of the categorical sentences using *is* [est in Latin] due to the peculiar characteristics of the Latin language. Due to the fact that Latin does not have a verb for “belong to” as Greek does, Boethius had no option but to adopt a new verb for “belong to”. This verb was *est* which means “is”. Boethius’s sentences then were in the following form:

(A) Every $B$ is $A$.

(I') Some $B$ is $A$.

(E') No $B$ is $A$.

(O') Some $B$ is not $A$.

Boethius’ categorical schemata, constructed accordingly to his categorical sentences are given in Table 3.1.

However, as mentioned earlier, Aristotle had used the word *belong*, instead of *is*. For this reason, it is said, Boethius was “accused of obscuring the theory of the syllogisms”, since his translation of *belong to* $is$, is claimed to make it unclear why the first figure (of Aristotle) was evident and was not in need of a proof ([8], p. 48). Obviously, the basic reason was the alteration of the order of the subjects and predicates in the premises. Because, the first figure, (see Table 2) heuristically manifests itself by the order of subjects and predicates. Patzig agrees with the claim that the first figure became less evident in Boethius’s translations and notes the following [10]:

The evidence of first figure arguments can be preserved in the *traditional* formulation (...). However, since traditional logic maintained the Aristotelian order of premises, the evidence which Aristotle rightly ascribed to the first figure syllogisms, and with it the distinction between perfect and imperfect arguments, was obscured.

<table>
<thead>
<tr>
<th>Table 7: Boethius’ Four Categorical Schemata</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
</tr>
<tr>
<td>$B \rightarrow A$</td>
</tr>
<tr>
<td>$C \rightarrow B$</td>
</tr>
<tr>
<td>$C \rightarrow A$</td>
</tr>
</tbody>
</table>
Furthermore, as stated in Marenbon’s treatise on Boethius ([8] p. 49), Boethius added a fourth conversion rule. Recall that in Section 2.2, we introduced the three rules that Aristotle gave. Boethius thought, just as a universal affirmative can be converted to a particular affirmative, a universal negative can be converted to a particular negative ([8] p. 49).

Boethius separated the cases for the valid syllogistic moods into two: the ones that need a demonstration, and the ones that do not. Agreeing with Aristotle, he calls the ones which do not need a demonstration perfect syllogisms. The remaining ones are named as imperfect syllogism. Boethius called the first figure syllogisms “indemonstrable” because they are “proven through themselves”. On the other hand, syllogisms in the second and the third figure was imperfect as they needed an “application of a conversion rule”. Boethius mentioned these points, according to Marenbon ([8], p. 48) in De Categoricis Syllogismis, 823A and, 817B and 812CD.

However, evaluated in an overall scale, Boethius can be considered as grasping the idea that the syllogism is different from the demonstration. Boethius wrote in De Categoricis Syllogismis which I took from [4], p. 314:

> We should not get anxious because here some of the propositions and conclusions are false, since it is not the truth of things we mean to discuss here but the connections of syllogisms, their moods and figures.

### 3.2 Hypothetical Syllogisms in Boethius

Boethius extended and enlarged the scholarly literature on HS, and “devoted a lot of his time to a tiresome but efficient work” on this ([4], p. 315). Furthermore, Martin remarked that ([9], p. 295), “At the beginning of De Hypotheticis Syllogismis, Boethius declares that he is the first to write in detail about hypothetical syllogisms. The Stoics, he tells us, produced nothing at all and among the Peripatetics only Theophrastus and Eudemus made even the barest beginnings.” That is essentially the reason why he was considered as the discoverer of HS for a long time. However, Martin further remarked that, Boethius’s claim that he was the first one wrote about HS in detail is “outrageously false”; because, “Boethius may well have known nothing of the range of Stoic logic.” ([9], p. 295)

According to Dürr in [5], Boethius “relates the theory of HS with Theophrastus and Eudemos”, and underlines that, Boethius claimed that “Aristotle wrote nothing” on HS. Boethius, furthermore, could not find any representation of HS in Latin scholars.

Boethius drew a distinction between the categorical and the hypothetical sentences, according to Lagerlund [7]. Boethius remarked that the categorical sentences involve a prediction while the hypothetical ones are conditionals. In De Topics Differentiis ([2], Book I, 1175), Boethius wrote:

> The parts of conditional propositions, which the Greeks call hypo-
theoretical\textsuperscript{5}, are simple propositions. The part of a conditional proposition that is said first is called the antecedent and the part that is said second is called the consequent.

In Book II, 1183, of [2], Boethius drew the aforementioned distinction between the categorical and the hypothetical sentences.

Of syllogisms, some are predictive - these are called categorical - and some are conditional - these we call hypothetical. Those made up of only predicative propositions are predicative (...). On the other hand, those whose propositions are connected by a condition are hypotheticals. For example: if it is a day, there is light; it is day; therefore, there is light. The first proposition contains the conditional that it is light just for this reason [namely], if it is day. So this syllogism is called hypothetical, that is, conditional.

However, the hypothetical sentences are constructed out of the categorical sentences linked by “if” or exclusive “or” as Boethius himself indicated in De Hypotheticis Syllogismis, I-1.5, 2.5 according to Marenbon ([8], p. 50). But Boethius devoted almost all of his attention to the hypothetical sentences constructed by “if” ([8], p. 50). Boethius only considered the hypothetical sentences from disjunctions at the end of the third book of De Hypotheticis Syllogismis (III-10.3-11.7), according to Marenbon ([8], p. 55).

For, Boethius, there were two kinds of the hypothetical sentences: simple and complex ones, and they were given in De Hypotheticis Syllogismis, I-3.5, I.5 according to Marenbon ([8], p. 51). Simple ones are of the form “If \(A\) is, then \(B\) is” whereas the complex ones are of the form “If \(A\) is, then, in case \(B\) is, \(C\) is too” or “If it is \(A\) then it is \(B\), then, if it is \(C\) then it is \(D\)”. Boethius came up with four different simple hypothetical sentences according to the affirmativity and the negativity of the antecedent and the consequent. He gave the four possible examples in ([2], Book I, 1176):

1. “If it is spherical, it is revolvable.” (Both affirmative)
2. “If the heaven is not spherical, it is not revolvable.” (Both negative)
3. “If it is square, it is not revolvable.” (Affirmative and negative respectively)
4. “If it is not spherical, it is stationary.” (Negative and affirmative respectively)

Boethius also considered the different types of complex HSs and different combinations of the hypothetical and the categorical syllogisms. He even considered classifying the modal syllogisms (which are many more), although we will not focus on modal syllogisms in this paper. But, in any case, he tried to give a complete table for the all possible types of HSs. Although the number is some

\textsuperscript{5}Emphasis is mine. - CB
hundreds, he “never tires of making all the combinations possible, enumerating all possible modes of the hypothetical reasoning” ([4], p.316).

Dumitriu gave Boethius’s examples of simple and complex hypothetical sentences taken from De Hypotheticis Syllogismis (from Book 2 and 3 of De Hypotheticis Syllogismis, but without proper page numbers) ([4], p.316):

1. “Si $A$ est, $B$ est.”
   (Simple hypothetical sentence)

2. “Si, cum $A$ est, $B$ est, cum sit $C$, est $D$.”
   (Complex sentence composed of two hypothetical sentences)

3. “Si $A$ est, cum sit $B$, est $C$.”
   (Complex sentence made up from one simple and one hypothetical sentence)

4. “Si, cum sit $A$, est $B$, erit et $C$”
   (Complex sentence hypothetical sentence is followed by a simple sentence)

Boethius’ analysis of HS started with his setting of terminology. Dumitriu in, [4], p.316, quoted the following passage from De Hypotheticis Syllogismis of Boethius.

Hypotheticos syllogismos quos latine conditionales vocamus alii quinque alii quatuor, alii tribus constare partibus arbitrantur ... quonian enim omnis syllogismus ex propositionibus textur, prima vel propositio vel sumptum vocatur secunda vero dicitur assumptio, ex his quae infertur conclusio nuncupatur.

“The hypothetical syllogisms, we call conditional in Latin, some deem them to consist of five parts, others of four, other of three ... and as any syllogism is made up of propositions [in order to discriminate them] the first is called either propositio or sumptum, the second is named assumptio, and the third inferred from the first two is called conclusio.”

(Translation is due to Dumitriu.)

This quotation well shows that how serious Boethius took the work of classification of syllogisms.

Boethius, moreover, gave the list of ten modes of hypothetical reasoning. The Latin list is taken from Dumitriu [4], p.317.

1. Si est $A$, est $B$
   Atqui est $B$
   Est igitur $B$

2. Si $A$ est, non est $B$
   Atqui est $A$
   Non est igitur $B$
3. Si non est $A$, est $B$
   Atqui non est $B$
   Est igitur $B$

4. Si non est $A$, non est $B$
   Atqui non est $B$
   Non est igitur $B$

5. Si est $A$, est $B$
   Atqui non est $B$
   Non est igitur $B$

6. Si est $A$, non est $B$
   Atqui est $B$
   Non est igitur $B$

7. Si non est $A$, est $B$
   Atqui non est $B$
   Est igitur $B$

8. Si non est $A$, non est $B$
   Atqui est $B$
   Est igitur $B$

9. Si non est $A$, est $B$
   Atqui est $B$
   Non est igitur $B$

10. Si non est $A$, est $B$
    Atqui est $A$
    Est igitur $B$

The following list of Boethius composed of twenty modes of hypothetical syllogisms composed of complex hypothetical sentences ([4], p.317).

1. Si sit $A$, cum sit $B$, est $C$.
   Atqui est $A$.
   Cum igitur sit $B$, est $C$.

2. Si sit $A$, cum sit $B$, non est $C$.
   Atqui est $A$.
   Cum igitur sit $B$, non est $C$.

3. Si sit $A$, cum non sit $B$, est $C$.
   Atqui est $A$.
   Cum igitur non sit $B$, est $C$.

4. Si sit $A$, cum non sit $B$, non est $C$.
   Atqui est $A$.
   Cum igitur non sit $B$, non est $C$. 
5. Si non sit $A$, cum sit $B$, est $C$.  
Atqui non est $A$.  
Cum igitur sit $B$, est $C$.  

6. Si non sit $A$, cum sit $B$, non est $C$.  
Atqui non est $A$.  
Cum igitur sit $B$, non est $C$.  

7. Si non sit $A$, cum $B$ non sit, est $C$.  
Atqui non est $A$.  
Cum igitur $B$ non sit, est $C$.  

8. Si non sit $A$, cum non sit $B$, non est $C$.  
Atqui non est $A$.  
Cum igitur non sit $B$, non est $C$.  

Atqui cum sit $B$, non est $C$.  
Igitur non est $A$.  

10. Si sit $A$, cum non sit $B$, non est $C$.  
Atqui cum sit $B$, est $C$.  
Igitur non est $C$.  

11. Si sit $A$, cum non sit $B$, est $C$.  
Atqui cum non sit $B$, non est $C$.  
Igitur non est $A$.  

12. Si sit $A$, cum sit $B$, non est $C$.  
Atqui cum non sit $B$, non est $C$.  
Igitur non est $A$.  

13. Si non sit $A$, cum non sit $B$, est $C$.  
Atqui cum non sit $B$, est $C$.  
Est igitur $A$.  

14. Si non sit $A$, cum sit $B$, non est $C$.  
Atqui cum non sit $B$, est $C$.  
Est igitur $A$.  

15. Si non sit $A$, cum non sit $B$, non est $C$.  
Atqui cum non sit $B$, est $C$.  
Est igitur $A$.  

16. Si non sit $A$, cum non sit $B$, non est $C$.  
Atqui cum non sit $B$, est $C$.  
Est igitur $A$.  

17. Si non sit $A$, cum sit $B$, est $C$.  
Atqui cum sit $B$, est $C$.  
Non est igitur $A$.  

13
18. Si non sit A, cum sit B, est C. 
Atqui est A. 
Cum igitur sit B, non est C.

19. Si non sit A, cum non sit B, est C. 
Atqui cum non sit B, est C. 
Non est igitur A.

20. Si non sit A, cum non sit B, est C. 
Atqui A. 
Cum non sit B, non est C.

Boethius claims that his book *De Hypotheticis Syllogismis* is the “fuller and more systematic treatment of the subject than anything he can find in Greek” (III-1.3.4) according to Marenbon ([8], p. 56). At this point Marenbon claims that the detailed “working out of the patterns of HS in Book II and III [of De Hypotheticis Syllogismis] is Boethius’s own” ([8], p. 56).

4 Conclusion

Regarding Aristotle’s works on logic, it is not unusual to claim that Aristotle extensively worked on categorical syllogisms, as we discussed earlier. In *Prior Analytics*, he gave proofs for many valid moods. However, when it comes to HS, Aristotle was not that active. Therefore, we cannot find a similar treatment for this subject in his works. Thus, it might be a reason for his students to improve Aristotle’s theory of syllogisms by filling the gaps in HSs.

In other words, Boethius could not base his treatment of HS on Aristotle’s works. As Boethius already emphasized in the Book I of *De Hypotheticis Syllogismis*, according to Marenbon ([8], p. 56), that the Peripatetic School only “initiated the start”, but the remaining and detailed treatise was due to Boethius himself.

Many arguments have been raised against Aristotle’s treatment of syllogisms ([10], many different pages) and maybe more can be said against Boethius’s interpretation of Aristotle’s works. We know that some modifications were inevitable for Boethius while translating Aristotle’s work. The very first reason was the differences between two languages that we already pointed out. Recall that the basic change in the order of subject and predicate caused a significant change in the figures.

However, we still can not be sure about the very fine details of Boethius’s commentaries. Because, some of the works that Boethius used extensively, such as Theophrastus’s works, did not survive. Therefore, once we knew the works of Theophrastus exactly, we could have decided to what extent Boethius developed the theory of HS by himself. Our only source for Theophrastus is secondary or tertiary resources that cites him. In a similar vein, maybe a more important reason why we are not so sure about Boethius’s logical commentaries is the lack
of the English translations of his works. Our main original resource in this paper was *De Topicis Differentiis*, which had an English translation. However, the logical enterprise embedded in this book was very limited to get a concrete idea about Boethius’ approach to HS. Unfortunately, the most important logical commentaries of Boethius, namely, *De Hypotheticis Syllogismis* and *De Categoricis Syllogismis* remains untranslated.

The impression we got from Boethius’s and Aristotle’s works on syllogisms is that the categorical syllogism is almost entirely developed by Aristotle himself, whereas the extensive and the detailed study of the hypothetical syllogism was carried out by Boethius, although he was not the first figure in the history who gave concise expositions on HS.

Further studies in this subject can be carried on, for instance, on the effects of Boethius’s translation of syllogisms in medieval era. As we remarked already, Boethius’s translations were the primer resource and textbook for Aristotelian logic during the early Medieval ages. Therefore, the scholars of those times had no other chance but to work from Boethius’s commentaries. So, the evident properties of Aristotelian figures were not then that visible. Another detailed study can be on the effects of Aristotle’s *the Organon* on Boethius’s original logic textbooks. We noted that, apart from his Aristotle commentaries, Boethius wrote several textbooks in logic. A detailed investigation might yield significant information on Boethius’s logic compared with the influence of Aristotle on Boethius.

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References


In the article we review modern studies in Boethius' treatise On the Hypothetical Syllogisms as exemplified by the works of Luca Obertello, Roberto Pinzani, Mauro Nasti de Vincentis and other historians of logic. We conclude that interpretation of R. Pinzani (2003) allows avoiding many difficulties encountered by the mathematical logicians during the analysis of the Boethius' doctrine of hypothetical syllogism. R. Pinzani proves that both the hypothetical and categorical syllogistic work on the same structural grounds. Unlike other researchers, Pinzani applies in his analysis the apparatus of the contemporary logic of predicates and combines categorical and hypothetical syllogistic with the general theory of proof developed in the Analytics of Aristotle. Hypothetical syllogisms are different from standard syllogisms and thus have their own rules. In a hypothetical syllogism the first premise (or major proposition) presents an uncertain condition ("if A, then B") or a problem ("either A or B"; "S and T cannot both be true") which must then be properly resolved by the second premise so that a valid conclusion can follow. The resolution of the problem is always in the form of affirmation or denial. In this article, the three types of hypothetical syllogism we will cover are the conditional syllogism, the disjunctive syllogism. The form of hypothetical syllogism is: â€œ if P, then Q. If Q, then R. Therefore, if P, then R. It may also be written as: P â†’ q q â†’ r ∴ p â†’ r. P, Q and R may represent any proposition, or any other formula (using Greek letters to represent formulae rather than propositions, we may also express modus tollens as α → β, β → γ α → γ). Examples of hypothetical syllogism. The following are examples of the hypothetical syllogism argument form: If it rains, we will not have a picnic. If we don't have a picnic, we won't need a picnic basket.