MEASUREMENT OF OUTPUT PRODUCTIVE EFFICIENCY AND OUTPUT TECHNICAL EFFICIENCY

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ABSTRACT
This theory is aimed at presenting how output productive efficiency measures are obtained using linear programming approach? The method is primarily an axiomatic approach. Estimation of output pure technical, scale, overall technical, allocative and overall productive efficiencies is explained by suitable numerical examples.

INTRODUCTION
In a path breaking article Farrell* introduced the concept of technical efficiency which is defined in terms of input reduction. The production process he assumed obeyed constant returns to scale, as the underlying production function is linear homogenous. His technical efficiency estimates are obtained using unit output isoquant as reference Technology. Farrell’s method was later extended by Koop**, whose reference technology was provided by a Frontier Production Function that admits any type of returns to scale, increasing, constant or decreasing. By Koop’s method can be implemented to measure technical efficiencies provided that the frontier production may be explicitly estimated.

***Timmer proposed linear programming approach to estimate an explicitly specified Frontier Production Function. The Frontier production function is a full Frontier in the sense that the output of each competing producer fall either below or on the production frontier.


Farrell’s method gives input oriented productive efficiencies. But his approach can be extended to outline output oriented productive efficiency measures such as output pure technical scale and overall Technical, allocative and overall productive efficiency measured.

For efficiency measurement, the chief tool is production function either parametric or non-parametric. A parametric frontier production may be fitted for data by either statistical estimation methods or methods of mathematical programming.

Schmidt*(1976), Schmidt and Lovell (1979) proposed statistical estimation procedures to estimate stochastic frontier production functions. These fitted frontier productions are implemented to measure technical and allocative efficiencies.

PIECE WISE LINEAR TECHNOLOGY - OUTPUT EFFICIENCY
The chief tool to measure the output efficiency is production function. A piece wise linear production function is linear approximation of a continuous once differentiable production frontier. Piece wise linear technology is axiomatic, based on the following axioms.

CONVEXITY
Let \( T = \{(x,u); x \text{ produces } u\} \) be a production possibility set.
\[
(x_i, u_i) \in T, \quad i = 1,2,3, \ldots k
\]
\[
\Rightarrow \left( \sum_{i=1}^{k} \lambda_i x_i, \sum_{i=1}^{k} \lambda_i u_i \right) \in T
\]
Where \( \lambda_i \geq 0 \), \( \sum_{i=1}^{k} \lambda_i = 1 \)

INEFFICIENCY
\[
(\tilde{x}, \tilde{u}) \in T \Rightarrow (x, u) \in T
\]
Where \( x \geq \tilde{x}, \ u \leq \tilde{u} \)

\[
(\tilde{x}, \tilde{u}) \in T \Rightarrow \text{There exists } \lambda_1, \text{ such that } (\tilde{x}, \tilde{u}) = (\sum \lambda_i x_i, \sum \lambda_i u_i), \ \sum \lambda_i = 1, \lambda_i \geq 0
\]
\[
\sum \lambda_i x_i \leq x
\]
\[
\sum \lambda_i u_i \geq u
\]
\[
\lambda_i \geq 0
\]
\[
\sum \lambda_i = 1
\]

MINIMUM EXTRACTION
\( T \) is the intersection of all \( T_a \)
\[
T = \cap_a T_a
\]
\[
T = \{(x,u); \sum x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0, \delta_i = 1\}
\]
The production possibility set \( T \) admits variable returns to scale.

RAY EXPANSION
\[
(\tilde{x}, \tilde{u}) \in T \Rightarrow \lambda (\tilde{x}, \tilde{u}) \in T, \ \lambda > 0
\]
\[
(\lambda \tilde{x}, \lambda \tilde{u}) = (\sum \lambda \lambda_i x_i, \sum \lambda \lambda_i u_i) \in T
\]
\[
\sum \lambda \lambda_i x_i \leq x
\]
\[
\sum \lambda \lambda_i u_i \geq u
\]
\[
\sum \lambda \lambda_i = 1
\]
\[
\delta_i \geq 0
\]
\[
T = \{(x,u); \sum \delta_i x_i \leq x, \sum \delta_i u_i \geq u, \delta_i \geq 0\}
\]
The production possibility set \( T \) admits constant returns to scale.

OUTPUT PURE TECHNICAL EFFICIENCY
\( \text{OPTE} = \max \theta \)
Such that
\[
\sum \lambda_i x_i \leq x_0
\]
\[
\sum \lambda_i u_i \geq u_0
\]
\[
\lambda_i \geq 0
\]
\[
\sum \lambda_i = 1
\]

Where \( x_i, u_i \) are the input and output vectors of \( \text{th} \) production unit, \( x_0, u_0 \) are the input and output vectors of the production unit whose efficiency is under evaluation.

OUTPUT OVERALL TECHNICAL EFFICIENCY
\( \text{OOTE} = \max \theta \)
Such that
\[
\sum \lambda_i x_i \leq x_0
\]
\[
\sum \lambda_i u_i \geq \theta u_0
\]
\[
\lambda_i \geq 0
\]

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OUTPUT SCALE EFFICIENCY
It is a derived measure
\[ OSE = \frac{OOTE}{OPE} \]

OVERALL OUTPUT PRODUCTIVE EFFICIENCY
To estimate overall output productive efficiency, potential revenue has to be estimated. Potential revenue can be obtained by solving the following linear programming problem.
\[ R(x_0,r_0) = \text{Max} \ r_0u \]
Such that
\[ \sum \lambda_i x_i \leq x_0 \]
\[ \sum \lambda_i u_i \geq \theta u_0 \]
\[ \lambda_i \geq 0 \]

OUTPUT OVERALL EFFICIENCY
\[ OOE = \frac{R(x_0,r_0)}{r_0u_0} \]
Where \( r_0u_0 \) is observed revenue and \( R(x_0,r_0) \) is potential revenue.

OUTPUT ALLOCATIVE EFFICIENCY
Output allocative efficiency is a derived measure
\[ OAE = \frac{OEE}{DOE} \]

NUMERICAL EXAMPLE
Given below are the inputs and outputs of five production units:

<table>
<thead>
<tr>
<th>UNIT</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( U_1 )</th>
<th>( U_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>25</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>20</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>35</td>
<td>10</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>E</td>
<td>40</td>
<td>25</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

OUTPUT PURE TECHNICAL EFFICIENCY
\[ \delta_P^D = \text{Max} \ \theta \]
Subject to
\[ 10 \lambda_1 + 20 \lambda_2 + 30 \lambda_3 + 35 \lambda_4 + 40 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 25 \lambda_2 + 20 \lambda_3 + 35 \lambda_4 + 25 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 20 \lambda_2 + 25 \lambda_3 + 10 \lambda_4 + 20 \lambda_5 \leq 30 \theta \]
\[ 40 \lambda_1 + 60 \lambda_2 + 70 \lambda_3 + 50 \lambda_4 + 90 \lambda_5 \leq 20 \theta \]
\[ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1 \]
\[ \lambda_i \geq 0 \]

Optimal solution of this linear programming problem gives output pure technical efficiency of the production unit D.

OUTPUT OVERALL TECHNICAL EFFICIENCY
\[ \delta^D = 1.5625 \]

OUTPUT OVERALL TECHNICAL EFFICIENCY
To find output overall technical efficiency of the production unit D, we solve,
\[ \theta^D = \text{Max} \ \theta \]
Such that
\[ 10 \lambda_1 + 20 \lambda_2 + 30 \lambda_3 + 35 \lambda_4 + 40 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 25 \lambda_2 + 20 \lambda_3 + 35 \lambda_4 + 25 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 20 \lambda_2 + 25 \lambda_3 + 10 \lambda_4 + 20 \lambda_5 \leq 30 \theta \]
\[ 40 \lambda_1 + 60 \lambda_2 + 70 \lambda_3 + 50 \lambda_4 + 90 \lambda_5 \leq 20 \theta \]
\[ \lambda_i \geq 0 \]

Solving LPP we obtain,
\[ \delta^D = 2.9166 \]

OUTPUT SCALE EFFICIENCY
Output scale efficiency is derived measure.
\[ \delta = \delta_1, \delta_2 \]
Where \( \delta_2 \) is output scale efficiency measure.
\[ \delta_2 = \delta_2^D = \frac{\theta^D}{\delta_2^D} = \frac{2.9166}{1.5625} = 1.8666 \]

Inference
\( \delta_0^D, \delta_1^D \text{ And } \delta_2^D \) Values reveals that the production unit D experiences heavy output losses due to pure technical, scale and hence overall technical efficiencies

OVERALL PRODUCTIVE EFFICIENCY
To compute the overall productive efficiency, primarily, positional revenue has to be estimated, by solving the following linear programming problem of production unit D.
\[ \pi = \text{Max} \ (0.35 \ u_1 + 0.35 \ u_2) \]
Such that
\[ 10 \lambda_1 + 20 \lambda_2 + 30 \lambda_3 + 35 \lambda_4 + 40 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 25 \lambda_2 + 20 \lambda_3 + 35 \lambda_4 + 25 \lambda_5 \leq 35 \]
\[ 10 \lambda_1 + 20 \lambda_2 + 25 \lambda_3 + 10 \lambda_4 + 20 \lambda_5 - u_1 \leq 0 \]
\[ 10 \lambda_1 + 15 \lambda_2 + 10 \lambda_3 + 12 \lambda_4 + 20 \lambda_5 - u_2 \leq 0 \]
\[ \lambda_i \geq 0 \]
\[ 0.35 \text{ and } 0.35 \text{ are unit selling prices of first and second outputs.} \]
Which is measured in some monetary units. The observed revenue is
\[ R = 0.35 \times 10 + 0.35 \times 12 \]
\[ R = 3.5 + 4.2 \]
\[ R = 7.7 \]

OVERALL PRODUCTIVE EFFICIENCY
\[ OPE = \frac{\pi}{R} = 4.5454 \]

ALLOCATIVE EFFICIENCY
This is another derived measure, defined as,
\[ OPE = \frac{OTE \times AE}{OPE} \]

For the production unit D allocative efficiency is,
\[ AE = \frac{4.5454}{2.9166} = 1.5584 \]

SUMMARY AND CONCLUSIONS
This study is aimed at presenting inefficiency problem of managerial economics on simple graphical terms first and extending them onto analytical grounds to measure various output productive effectiveness such as output pure technical, scale, overall technical, overall productive and scale efficiencies. Numerical illustrations are given to have clear insight into the problem of inefficiency and its measurement.

BIBLIOGRAPHY

• Kopp, R.J., “The measurement of productive efficiency A Reconsideration” the quarterly Journal of Economics, 96 (1981), 477-503
• Subbaratri Reddy, C., Productivity and Technological Change in the selected Indian Industries, Unpublished Ph.D., Thesis S.V.University, India, 1983
Productive efficiency (or production efficiency) is a situation in which the economy or an economic system (e.g., a firm, a bank, a hospital, an industry, a country, etc.) could not produce any more of one good without sacrificing production of another good and without improving the production technology. In other words, productive efficiency occurs when a good or a service is produced at the lowest possible cost. In simple terms, the concept is illustrated on a production possibility frontier (PPF).